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DIGITAL IMAGE RESTORATION USING CONDITIONAL MARKOV MODELS.(U)

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DIGITAL IMAGE RESTORATION USING CONDITIONAL MARKOV MODELS

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ABSTRACT

We are interested in developing digital image restoration algorithms using a class of spatial interaction models known as conditional Markov models. Our approach is to represent the images by Markov models on toroidal lattices and develop minimum mean square error (MMSE) restoration algorithms. The algorithms are non-recursive in structure and due to the underlying representation on toroidal lattices can be implemented using FFT algorithms.

We give two types of algorithms. First we assume that a prototype of the original image is available and develop algorithms for restoration of degraded images. The degradation is assumed to be due to a known space invariant, non-separable, periodic point spread function and additive white noise. Secondly, we consider the more general situation when a prototype image is not available and give algorithms for MMSE filtering of noisy images. Experimental results are given for the above cases.

The support of the U.S. Air Force Office of Scientific Research under Grant AFOSR-77-3271 is gratefully acknowledged. The author is grateful to Profs. R.L. Kashyap and A. Rosenfeld for helpful discussions. The help of Sherry Palmer in preparing this paper is also acknowledged.

1. Introduction

The restoration of degraded images has many fields of application, including space imagery and biomedical images [1,2]. The literature on image restoration is too enormous to be listed in detail. There are many different methodologies such as MMSE restoration, maximum a posteriori probability restoration, and maximum entropy restoration [3]. We are concerned with MMSE methods in this paper.

Suppose $\underset{\sim}{y}$ and $\underset{\sim}{x}$ represent the lexicographically ordered arrays of the original and degraded image, related by

$$\underset{\sim}{x} = \underset{\sim}{H}\underset{\sim}{y} + \underset{\sim}{\eta} \tag{1.1}$$

In (1.1), $\underset{\sim}{H}$ is the block-circulant matrix corresponding to the blur caused by a non-separable, space invariant point spread function (PSF) and $\underset{\sim}{\eta}$ is signal independent additive white noise of variance γ. Then it is well known [1,p.136] that $\hat{\underset{\sim}{y}}$, the MMSE estimate of $\underset{\sim}{y}$, can be written as

$$\hat{\underset{\sim}{y}} = \underset{\sim}{Q}_y \underset{\sim}{H}^T (\underset{\sim}{H} \underset{\sim}{Q}_y \underset{\sim}{H}^T + \gamma I)^{-1} \underset{\sim}{x} \tag{1.2}$$

where $\underset{\sim}{Q}_y$ is the covariance matrix of $\underset{\sim}{y}$.

There are two problems to be considered in the computation of $\hat{\underset{\sim}{y}}$, namely, the determination of the covariance matrix $\underset{\sim}{Q}_y$ and the inversion of the matrix in (1.2). Suppose we assume that the prototype of the original is available. Then $\underset{\sim}{Q}_y$ can be evaluated by making appropriate assumptions about the correlation structure of the prototype image. For instance, one may consider

a separable correlation function [4], whose parameters are chosen arbitrarily. In [1, Ch. 7], $\hat{\underset{\sim}{y}}$ of (1.2) is equivalently written in terms of the spectral density function (SDF) of the prototype, which is often estimated from the periodogram.

The second problem is concerned with the matrix inversion in (1.2). For an image of size MxM, $\underset{\sim}{Q}_y$ is of dimension M^2xM^2 and it is not uncommon to have M = 128 in typical restoration applications. Thus some assumptions have to be made to reduce the computational load. In [1, Ch. 7] the block Toeplitz covariance matrix $\underset{\sim}{Q}_y$ is approximated by a block-circulant matrix. Since block-circulant matrices possess an eigenfunction expansion in terms of Fourier vectors, FFT computations are used for the implementation of (1.2). In [4], appropriate assumptions are made regarding the model representation for $\underset{\sim}{y}$, so that $\underset{\sim}{Q}_y$ has a symmetric, tri-diagonal Toeplitz form. Since, symmetric, tri-diagonal matrices are diagonalized by sine transforms, fast algorithms for the implementation of (1.2) have been developed. Another approach [5,6] is to assume an underlying unilateral model for $\underset{\sim}{y}$ displayed as a state space model. Then $\hat{\underset{\sim}{y}}$ could be computed by recursive algorithms of the Kalman type.

In this paper we present a class of MMSE restoration algorithms which do not make restrictive assumptions such as causality, isotropy, or separable correlation function. We assume that the images are represented by a class of models

known as spatial interaction models. One of the important characteristics of an image is the statistical dependence of the gray levels at a lattice point on those at its neighbors. Spatial interaction models (familiarly known as random field models) characterize this statistical dependency by representing $y(i,j)$, the gray level at location (i,j), as a linear combination of the gray levels at the neighboring pixels and additive noise. We characterize this dependency using a neighbor set N which is a finite collection of pairs of integers (i_1,j_1), not including $(0,0)$, such that $y(i,j)$ is dependent on $\{y(i + i_1, j + j_1), (i_1,j_1) \in N\}$. Among the spatial interaction models we are primarily interested in the class of conditional Markov (CM) models [7-12]. We represent either the given degraded image, $\underset{\sim}{x}$ in (1.1), or the original $\underset{\sim}{y}$, by appropriate CM models and formulate the MMSE restoration problem. The representation on toroidal lattices leads to a covariance matric $\underset{\sim}{Q}_x$ that is block-circulant, leading to fast implementation of filters using FFT algorithms.

Our contributions can be summarized as follows: First, the CM models used in this paper are more general than the models used in [4]. We do not make either separability or isotropy assumptions. The recursive algorithms given in [5-6] are exact only when the SDF of the underlying CM model factorizes.

In general the SDFs of CM models do not factorize. For these models, the MMSE criterion naturally leads to non-recursive algorithms and the recursive algorithms are only approximate since these algorithms use unilateral models whose SDF is only an approximation to the SDF of the true CM model.

Secondly, we give results of filtering noisy images using CM models without requiring the availability of an original image [9,13] or knowledge of the additive noise variance γ, an omnipresent assumption in the literature on MMSE image restoration.

The organization of the paper is as follows: in Section 2, we give a brief description of the representation of images using CM models. In Section 3, we develop two types of restoration algorithms and give experimental results. Finally, discussion is presented in Section 4.

2. Representation of Images Using Conditional Markov Models

Suppose we are given an image $\{y(s),\ s = (i,j) \in \Omega\}$, $\Omega = \{s = (i,j),\ 1 \le i,j \le M\}$ where $y(s)$ is the gray level of the cell s. We partition the finite lattice Ω into mutually exclusive and totally inclusive subsets Ω_I, the interior set, and Ω_B, the boundary set, defined as

$$\Omega_I = \{s = (i,j):\ s \in \Omega \text{ and } (s + s') \in \Omega\ \forall s' \in N\}$$

and

$$\Omega_B = \{s = (i,j):\ s \in \Omega \text{ and } (s + s') \notin \Omega \text{ for at least one } s' \in N\}.$$

The toroidal lattice representation relevant to restoration of finite images [8-10,12,13], is arrived at by assuming the following representation for $\{y(s)\}$:

$$y(s) = \sum_{s' \in N} \theta_{s'}\, y(s + s') + \sqrt{\nu}\, e(s),\quad s \in \Omega_I \qquad (2.1)$$

and

$$y(s) = \sum_{s' \in N} \theta_{s'}\, y_1(s + s') + \sqrt{\nu} e(s),\quad s \in \Omega_B \qquad (2.2)$$

where $y_1(s)$ with $s = (k,\ell) = y\,[s + (i,j)]$ if $s + (i,j) \in \Omega$ and $(i,j) \in N$ (2.3)

$= y[(k + i - 1) \bmod M + 1,\ (\ell + j - 1) \bmod M + 1]$ if $s + (i,j) \notin \Omega$ and $(i,j) \in N$ (2.4)

In (2.1), N is a symmetric neighbor set in that if $s' \in N$, then $-s' \in N$. Further $\theta_s = \theta_{-s}$. The noise sequence $e(s)$ is independent of $y(r)$, $s \neq r$. In particular, $E(e(s)|y(r)) = 0$, $r \neq s$ (2.5) which implies

$$E(y(s)|y(r),\ r \in \Omega,\ r \neq s)$$
$$= \sum_{s' \in N} \theta_{s'}\, y_1(s + r) \qquad (2.6)$$

Equation (2.6) implies that $\{y(\cdot)\}$ is Markov [7] with respect to the symmetric neighbor set N. The noise sequence $\{e(s)\}$ is correlated with the correlation structure in (2.7), which follows from (2.5) or (2.6):

$$E(e(s)\ e(s')) = \begin{cases} 1 & s = s' \\ -\theta_{s-s'}, & s - s' \in N \\ 0 & \text{otherwise} \end{cases} \tag{2.7}$$

To ensure stationarity, (2.8) should be valid [9]:

$$\mu_s = (1-2\ \underset{\sim}{\theta}^T \underset{\sim}{\phi}_s) > 0 \tag{2.8}$$

where

$$\underset{\sim}{\phi}_s = \text{Col. } [\cos \frac{2\pi}{M} [(s-1)^T s'],\ s' \in N_s]$$

Here N_s is a proper subset of N such that

$$s \in N_s \Rightarrow -s \notin N_s$$

$$N_s \cup N_{\bar{s}} = N$$

and $\quad N_{\bar{s}} = \{s' : -s' \in N_s\}$

Using lexicographically ordered notation for the arrays $\{y(s)\}$ and $\{e(s)\}$, (2.1) - (2.2) can be written in matrix/vector format as

$$A(\underset{\sim}{\theta})\ \underset{\sim}{y} = \sqrt{\nu}\underset{\sim}{e} \tag{2.9}$$

where $A(\underset{\sim}{\theta})$ is a symmetric block-circulant matrix. As $A(\underset{\sim}{\theta})$ has an inverse by (2.8),

$$\underset{\sim}{y} = \sqrt{\nu} A(\underset{\sim}{\theta})^{-1} \underset{\sim}{e} \tag{2.10}$$

and the covariance matrix

$$\underset{\sim}{Q}_y = E(\underset{\sim}{y}\underset{\sim}{y}^T) = \ \nu A^{-1}(\underset{\sim}{\theta}) \tag{2.11}$$

is also block-circulant with eigenvalues ν/μ_s, $s \in \Omega$. By different

choices of N we account for different filter structures. For instance $N = \{(0,1),(1,0),(-1,0),(0,-1)\}$ corresponds to the first order Markov model [7].

3. Restoration Schemes Using CM models

In developing restoration algorithms we consider the following two cases.

Case (i): A CM model with parameters $(\underset{\sim}{\theta}, \nu)$ is assumed for the original image $\underset{\sim}{y}$. It is assumed that a prototype of the original image is available such that $(\underset{\sim}{\theta}, \nu)$ can be estimated. A degraded image $\underset{\sim}{x}$ related to $\underset{\sim}{y}$ as in (1.1) is given and it is required to develop algorithms for computing $\hat{\underset{\sim}{y}}$.

Case (ii): We are given a noisy image only, with no prototype of the original being available. It is required to develop filtering algorithms.

3.1.a Restoration Using CM Models for $\underset{\sim}{y}$ (Case i).

Assume that the original image $\underset{\sim}{y}$ is represented by (2.1) and (2.2) and that $\underset{\sim}{x}$ is related to $\underset{\sim}{y}$ as in (1.1). Then the MMSE estimate $\hat{\underset{\sim}{y}}$ is given in (1.2).

Let $\underset{\sim}{f}_s$ = Col. $[\underset{\sim}{t}_j, \lambda_i \underset{\sim}{t}_j, \ldots, \lambda_i^{M-1} \underset{\sim}{t}_j]$, with $s = (i,j)$

$\underset{\sim}{t}_s$ = Col. $[1, \lambda_j, \lambda_j^2, \ldots, \lambda_j^{M-1}]$, M vector

$\lambda_i = \exp[\sqrt{-1}\lambda_0 (i-1)], \; \lambda_0 = 2\pi/M$

be the M^2 Fourier vectors and let $\bar{h}_s$, $s \in \Omega$ be the eigenvalues of $\underset{\sim}{H}$. Then due to the underlying toroidal lattice representation, $\hat{\underset{\sim}{y}}$ can be computed as

$$\hat{\underset{\sim}{y}} = \frac{1}{M^2} \sum_{\Omega} \underset{\sim}{f}_s (\nu \bar{h}_s^* / (\nu||\bar{h}_s||^2 + \gamma \mu_s) \underset{\sim}{f}_s^{*T} \underset{\sim}{x} \qquad (3.1)$$

where * denotes the complex conjugate operator. The computation

of $\hat{\underset{\sim}{y}}$ can be done by using FFT algorithms and the computational complexity is $O(M^2 \log M)$. The computing scheme in (3.1) has a general structure without involving any assumptions such as isotropy or separable correlation structure. Equation (3.1) is characterized by ν, $\underset{\sim}{\theta}$ and γ. It is assumed that γ is known. The parameters $\underset{\sim}{\theta}, \nu$ are assumed arbitrarily [4] or estimated from the prototype of the original. There are several methods of estimating $(\underset{\sim}{\theta}, \nu)$ for CM models. The coding method in [11] is very simple in principle and yields consistent but very inefficient estimates. The ML method in [12] yields asymptotically consistent and efficient estimates but is computationally unattractive except for isotropic models. A simple consistent estimate whose efficiency lies between the coding and ML methods, given in [14], is used to estimate $\underset{\sim}{\theta}$ and ν.

A numerical figure of merit for restoration algorithms is the expected minimum error ω. From [1,pp. 140],

$$\omega = \frac{1}{M^2} \operatorname{Tr}[\underset{\sim}{Q}_y - \underset{\sim}{L}\, \underset{\sim}{H}\, \underset{\sim}{Q}_y] \tag{3.2}$$

where

$$\underset{\sim}{L} = \underset{\sim}{Q}_y \underset{\sim}{H}^T (\underset{\sim}{H}\, \underset{\sim}{Q}_y\, \underset{\sim}{H}^T + \gamma I)^{-1} \tag{3.3}$$

Using the fact that $\underset{\sim}{Q}_y$ and $\underset{\sim}{H}$ are block-circulants, with eigenvalues ν/μ_s and h_s, $s \in \Omega$,

$$\omega = \frac{1}{M^2} \sum_{\Omega} \frac{\nu\gamma}{\nu||\bar{h}_s||^2 + \gamma\mu_s)} \tag{3.4}$$

3.1.b <u>Experimental Results</u>: To illustrate the use of (3.1), the following experiment was done. A 64x64 window of the original USC girl image was used. The parameters corresponding to different neighbor sets N_s were computed. The

image was blurred by using the PSF

$$h(k,\ell) = \frac{0.4}{\pi} \exp\{-0.4(k^2+\ell^2)\}, 0 \le k, \ell \le 2, \qquad (3.5)$$

and Gaussian noise of signal to noise ratio (defined as the ratio of signal variance to noise variance) of 5 was added to the blurred image. The results of implementing (3.1) are in Fig. 1 for SNR = 5 and in Fig. 2 for SNR = 1. The details of the models used are in the second column of Table I. The restored images corresponding to neighbor sets N_{s3} are good both for the SNR = 5 and 1 cases. But the restored images corresponding to neighbor sets N_{s1}, N_{s2} are not as good for SNR = 1 as they are for SNR = 5 blurred images. Filters of relatively large order were used for this image since lower order CM models were non-stationary.

To get some idea as to the amount of theoretical improvement that is possible when using different neighbor sets, numerical values of the expected error ω were computed using (3.4). The quantity g defined as

$$g = 10\log \frac{\text{MSE between original and degraded}}{\omega} \qquad (3.6)$$

is tabulated in Table I for SNR = 5 and 1 for the blur in (3.5). The values of the numerator of (3.6) are 511.41 and 1910.1 respectively.

Note that thus far we have assumed that the additive noise sequence η is white. The case when η is colored [4] can be considered by replacing γI in (1.2) by $\underset{\sim}{C}$ where $\underset{\sim}{C} = E(\underset{\sim}{\eta}\,\underset{\sim}{\eta}^T)$ and is known.

3.2.a <u>Restoration using CM models for x</u>: In practice, the CM models of the original image are not known and only the degraded image is available. It is required to obtain $\hat{\underset{\sim}{y}}$ using only the given degraded image. Suppose that degradation is due to additive noise only, i.e., $\underset{\sim}{x}$ is related to $\underset{\sim}{y}$ by

$$\underset{\sim}{x} = \underset{\sim}{y} + \underset{\sim}{\eta} \qquad (3.7)$$

Assume that $\underset{\sim}{x}$ is represented by the CM model

$$x(s) = \sum_{s' \in N} \theta'_{s'} x(s + s') + \sqrt{\nu'}\, e(s), \; s \in \Omega_I \qquad (3.8)$$

and

$$x(s) = \sum_{s' \in N} \theta'_{s'} x_1(s + s') + \sqrt{\nu'}\, e(s), s \in \Omega_B \qquad (3.9)$$

where $x_1(\cdot)$ obeys an equation similar to $y_1(\cdot)$ in (2.3) and (2.4). The covariance matrix $\underset{\sim}{Q}_x$ of $\underset{\sim}{x}$ under the transformation in (3.7) is

$$\underset{\sim}{Q}_x = \underset{\sim}{Q}_y + \gamma I \qquad (3.10)$$

or

$$\underset{\sim}{Q}_y = \underset{\sim}{Q}_x - \gamma I \qquad (3.11)$$

By substitution of (3.11) in (1.2) and letting H = I, we obtain the MMSE estimate

$$\hat{\underset{\sim}{y}} = (\underset{\sim}{Q}_x - \gamma I)(\underset{\sim}{Q}_x)^{-1} \underset{\sim}{x} \qquad (3.12)$$

Since $\underset{\sim}{Q}_x$ is block-circulant with eigenvalues $\nu'/\mu'_s, s \in \Omega$, $\hat{\underset{\sim}{y}}$ can be computed as

$$\hat{\underset{\sim}{y}} = \frac{1}{M^2} \sum_{\Omega} \underset{\sim}{f}_s \frac{(\nu' - \gamma\mu'_s)}{\nu'} \underset{\sim}{f}_s^{*T} \underset{\sim}{x} \qquad (3.13)$$

Equation (3.13) yields an optimal $\hat{\underset{\sim}{y}}$ if the parameters $\underset{\sim}{\theta}'$, ν' and γ are known exactly. In practice they are not known and are estimated from the given noisy image. As discussed before $\underset{\sim}{\theta}'$, ν' can be estimated using the estimation scheme in [14] and

the noisy image. A scheme which has been found to be of some practical use [1, pp. 144] in estimating γ is to use the steady state component of the SDF of the noisy image as an estimate. An estimate $\hat{\gamma}$ of γ is obtained for 64×64 images using

$$\hat{\gamma} = \frac{1}{16} \sum_{\Omega_u} S_x(s), \quad \Omega_u = \{s = (i,j), 30 \le i,j \le 33\} \qquad (3.14)$$

where $s_x(s) = \nu'/\mu'_s$, $s \in \Omega$, is the two-dimensional discrete SDF of $\underset{\sim}{x}$ under the representation (3.8) - (3.9).

3.2.b Experimental Results: The original girl's face in Fig. 1 was corrupted by additive Gaussian noise of SNR = 5. It was assumed that only the noisy image was available. CM models having different neighbor sets N_s were fitted to the noisy image. The filtered images for the SNR = 5 noisy image obtained using (3.13) are shown in Fig. 3. The details of the models used are given in Table II. Similar results for SNR = 1 are given in Fig. 4. The filtered images of the SNR = 5 noisy image corresponding to neighbor sets N_{s1}, N_{s5} and N_{s6} are good. The improvement is noticeable in the filtered images of the SNR = 1 noisy image corresponding to neighbor set N_{s1}.

The parameter γ was estimated using the scheme discussed in Section 3.2.a. The best estimate of γ was obtained for the SNR = 5 image; it was 378.74 (true value being 393.01) using neighbor set N_{s1}, and for a 0 db noisy image the estimate was 1917.9 (the true value being 1965.05) using neighbor set N_{s6}.

Actual values of MSE between the original and the filtered images in Fig. 3 and 4 were computed. The MSE corresponding to the best filtered image in Fig. 3 was 91.08 using the neighbor set N_{s1} with the error between the original and noisy image being 378.53, roughly a reduction in error by a factor of 4. Similar test results for the SNR = 1 image are 321.79 and 1728.5 using N_{s6}, representing a reduction in noise by a factor of 5. For the details of estimates of γ and MSE for other CM models see Tables II and III.

4. Discussion

We have considered some MMSE algorithms for the restoration of degraded images using CM models. Assuming that a prototype of the original image is available, an algorithm for restoring a blurred and noisy image has been given together with an expression for the expected minimum error. A computing scheme for the filtering of noisy images, when the prototype of the original image is not available, has been given along with experimental results.

The algorithms are fast, non-recursive, and are valid for any arbitrary neighbor sets N_s. We have used a toroidal lattice representation for the images, which might appear restrictive. For the values of M considered here it can be shown [9,15] that the second-order properties of a CM model on a toroidal lattice are very nearly equal to those of the corresponding CM model on an infinite lattice. To illustrate this the normalized autocorrelation function (ACF) of an isotropic CM model with $\theta = .24$ and $N_s = \{(1,0),(0,1)\}$ was evaluated at a few lower lags using the toroidal assumption and compared with the corresponding quantities for the infinite lattice model available in [16, Table 1]. Denoting by $\rho_{k,\ell}$ the normalized ACF at lag (k,ℓ) we have, for the ACF computed with infinite lattice model, $\rho_{1,0} = .434$, $\rho_{1,1} = .295$, $\rho_{0,2} = .220$ and $\rho_{2,1} = .180$. The corresponding quantities computed for the

toroidal lattice are $\rho_{1,0} = .4329$, $\rho_{1,1} = .2935$, $\rho_{0,2} = .2181$ and $\rho_{2,1} = .1785$ which are numerically close.

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Table I. Performance bounds in decibels as predicted by various CM models for the PSF in (3.5) and M = 64.

Number	Neighbor set N_s	SNR = 7 db	SNR = 0 db
N_{s1}	$\{(1,0),(0,1),(1,1),(1,-1),(0,2),(2,0)\}$	5.72	7.44
N_{s2}	$\{(0,1),(1,0),(1,-1),(1,1),(0,2),(2,0),(-2,1),(2,1),(-1,2),(1,2)\}$	5.27	6.66
N_{s3}	$\{(1,0),(0,1),(1,-1),(1,1),(0,2),(2,0),(-2,1),(2,1),(-1,2),(1,2),(2,2),(-2,2)\}$	5.45	8.21

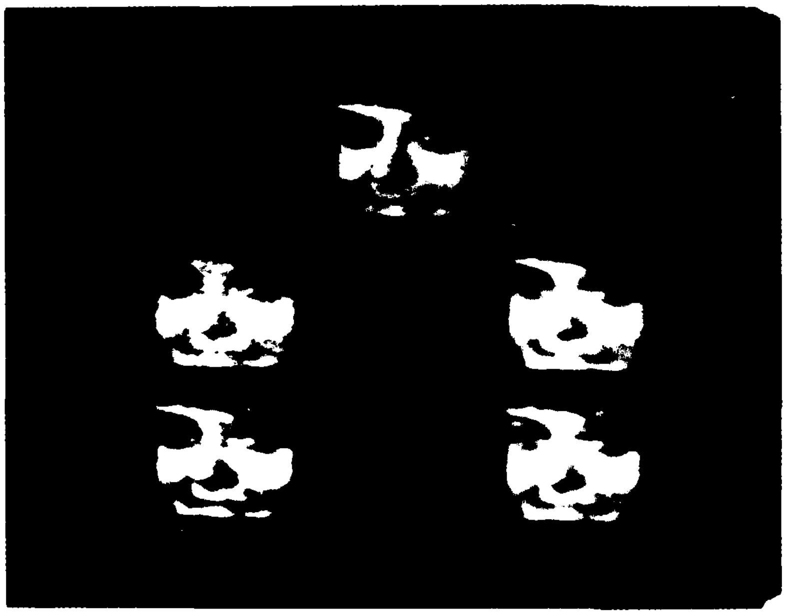
Table II. The estimates of γ obtained using (3.14). The true values of γ are 393.01 (SNR =5) and 1965.05 (SNR = 1).

Neighbor set	SNR = 5	SNR = 1
$N_{s4} = \{(0,1),(1,0)\}$	264.63	1183.9
$N_{s5} = \{(0,1),(1,0),(1,1)\}$	318.62	1493.0
$N_{s6} = \{(0,1),(1,0),(1,1),(1,-1)\}$	358.79	1917.9
N_{s1}	378.74	2185.1

Table III. Mean squared error between the original and filtered images. The errors between the original and noisy pictures were 378.53 and 1728.5 for SNR = 7 db and 0 db respectively.

Neighbor Set	SNR = 7 db	SNR = 0 db
N_{s4}	129.35	499.90
N_{s5}	103.81	332.21
N_{s6}	93.28	321.79
N_{s1}	91.08	339.58

(a)



(b) (c)

(d) (e)

Fig. 1. Restoration of image containing blur (3.5) and Gaussian noise (SNR=5).
a) Original
b) Blurred
c-e) Restored images using N_{s1}, N_{s2}, N_{s3}.

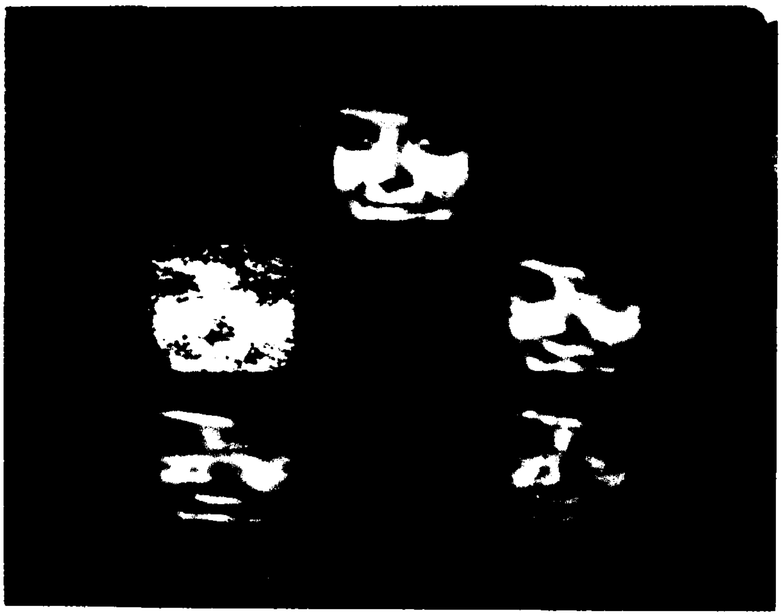


Fig. 2. Analogous to Fig. 1 with SNR=1.

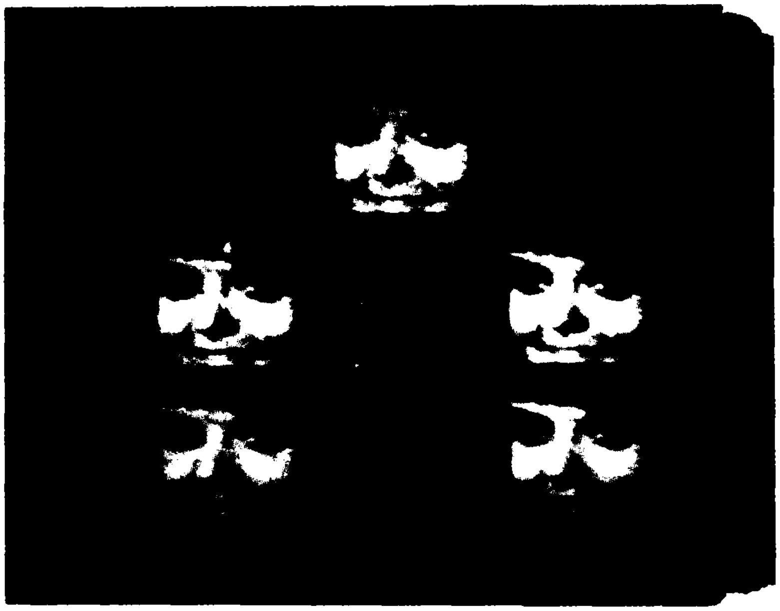


Fig. 3. Filtering of noisy image when prototype of original is not available.
a) Noisy image (SNR=5).
b-e) Filtered images using N_{s4}, N_{s5}, N_{s6}, N_{s1}.

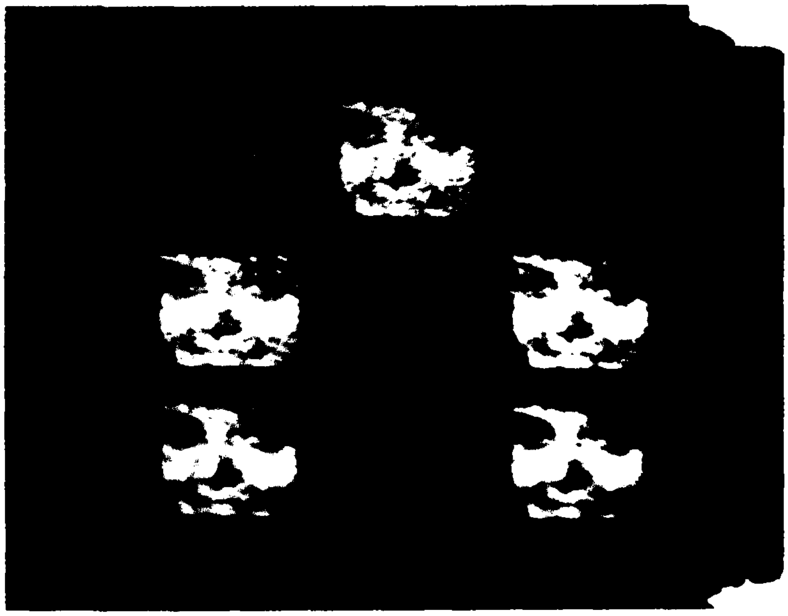


Fig. 4. Analogous to Fig. 3 with SNR=1.

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4. TITLE (and Subtitle) DIGITAL IMAGE RESTORATION USING CONDITIONAL MARKOV MODELS	5. TYPE OF REPORT & PERIOD COVERED INTERIM
	6. PERFORMING ORG. REPORT NUMBER TR-1027
7. AUTHOR(s) R. Chellappa	8. CONTRACT OR GRANT NUMBER(s) AFOSR-77-3271
9. PERFORMING ORGANIZATION NAME AND ADDRESS Computer Science Center, University of Maryland, College Park, MD 20742	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS PE 61102F 2304/A2
11. CONTROLLING OFFICE NAME AND ADDRESS Math & Info. Sciences, AFOSR/NM, Bolling AFB, Washington, D. C. 20332	12. REPORT DATE March 1981
	13. NUMBER OF PAGES 22
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Image processing
Image models
Image restoration
Markov models

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We are interested in developing digital image restoration algorithms using a class of spatial interaction models known as conditional Markov models. Our approach is to represent the images by Markov models on toroidal lattices and develop minimum mean square error (MMSE) restoration algorithms. The algorithms are non-recursive in structure and due to the underlying representation on toroidal lattices can be implemented using FFT algorithms.

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

We give two types of algorithms. First we assume that a prototype of the original image is available and develop algorithms for restoration of degraded images. The degradation is assumed to be due to a known space invariant, non-separable, periodic point spread function and additive white noise. Secondly, we consider the more general situation when a prototype image is not available and give algorithms for MMSE filtering of noisy images. Experimental results are given for the above cases.